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Supercomputer Simulation and Approximation for Surface Forms of the Normal-Field Instability in Magnetic Liquids Using Particle Swarm Optimization Approach and Griewank Function

Abstract: In this short note we consider the way in which a useful approximation of ferrofluid liquid surface can be obtained with a very simple mathematical function. Many ideas and results of this work can be extended to the case of biological morphogenesis, including biomorphic pattern morphogenesis by ferromagnetic liquids. However, most of the materials presented in this article deal with simple ferrofluid forms. The approximation is not applicable for different ferrofluid surface morphologies, because this possibility can be used only in very special applications, where the solution (or approximation) process is complicated by the presence of some more patterning effects. We shall not consider this complicated question in this article. We only pay attention to the fact that some ferrofluid patterning mechanisms admit a simple geometrical interpretation in the frame of particle swarm optimization theory.

Keywords: ferrofluide, particle swarm optimization, PSO, nanoparticles, Griewank function, ferromagnetic liquid

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Introduction

It is well known that ferrofluids are colloidal liquids [1] formed of nano-ferromagnetic particles [2], suspended in a liquid carrier, which easily become magnetized in the presence of magnetic field [3]. When a paramagnetic fluid is exposed to an intense vertical magnetic field, the surface demonstrates normal-field instabilities, such as regular peak and valley patterns, because these relief forms are energetically profitable [4]. The formation of the surface relief deformations increases the surface free energy and the potential energy, associated with the gravitational field of the liquid, and reduces the work of magnetic force or torque on the re-orientation of the magnetic dipole moment vector.

However, ferrofluids provide not only physico-chemical opportunities, but they are also useful in morphogenesis simulation [5]. It is also important that some ferrofluid patterning mechanisms admit a simple geometrical interpretation and approximation. For many applications this is all that is required for ferrofluid surface properties illustration. In many cases it is necessary to use complex approximation methods because the solution process is complicated by the presence of some nonlinear effects [6] and ferrofluid patterning mechanisms [7]. This is obviously a more complex problem than the usual determination of a simple liquid dynamics [8]. It is evident that a more refined argument is required for this problem and it is important to understand the nature of these approximations, as well as the ferrofluid dynamics nature [9].

Let us begin with defining more carefully what we mean by physical nature in this case. For the purpose of analysis we shall assume that ferrofluid is a particle manifold [10]. Incidentally, it is to be noted that magnetic particle manifold under magnetic field is an ordered manifold [11]. Moreover, it is easily possible to demonstrate that ferrofluid magnetic particle assemblies in strong magnetic fields are cooperative dynamic sets [12] or dynamic manifolds [13, 14]. Practitioners (like ourselves) rarely worry about mathematical rigor, but if necessary this can be proved without difficulties. In this short note we shall not pay any attention to mathematical aspects of this problem.

It is important to understand how to apply the concept of physical similarity [15-21] for appropriate approximation selection in ferrofluid hydrodynamics. This method is applicable for a large class of physical systems, but we shall not consider this very extensive question. For example, an application of collective particle dynamics laws gives very simple explanation for particle swarm optimization [22-24] applicability in approximation of cooperative ferrofluid nanoparticle dynamics in a liquid carrier. This is also in accordance with experimental physical observations [25, 26]. To a lesser extent similar considerations hold for other ferroparticle systems without strong magnetic fields, but we shall not attempt to give significance to particle swarm optimization approaches in some less ordered (and, consequently, less swarming) particle systems.

Thus we should confine ourselves to finding correct (morphologically similar) approximation for surface forms of the normal-field instabilities [27] visualized as regular peak and valley patterns using the function choice within particle swarm optimization. Henceforth, we shall not attempt to distinguish between normal-field instability forms and forms of ferrofluid surface under normal-field instability, because it is possible to establish one-to-one mapping between them. A problem that we should inevitably face while using this concept is one-to-one mapping between ferrofluid surface deformation parameters and magnetic field parameters. Of course this approach applies only if we know all of them, but for the present approximation, however, we neglect most of the second order effects. Difficulties arise as soon as we try to approximate ferrofluid surface forms using ab initio approach, but in the first-order approximation we may ignore some technical complications. Nevertheless, attention needs to be paid to morphological similarity between approximation visualization form and elementary ferrofluid surface deformation.

Results

In this section we illustrate the application of the above approach. For example, a useful approximation is obtained by a function from particle swarm optimization area, known as Griewank function [28, 29]:

$$f\left(x\right)=1+\sum_{i=1}^{n}\frac{x_i^2}{4000}-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)$$

Now we should consider an important question: how good this approximation is? This approximation is valid whenever an obvious visual isomorphism exists between this approximation and surface forms of a ferrofluid liquid. The nature of the approximation is illustrated in Fig. 1 and the photo of ferrofluid surface with the normal-field instability is given in Fig. 2. A similar relationship exists between some physical objects only when one of them approximates the other. A better approximation can be obtained by numerical parameter choice, but in our calculations we used particle swarm optimization approach, so such numerical approaches are of little significance in our case [30]. From the arguments completely analogous to those presented in the previous chapter we conclude that morphological similarity in this case corresponds to the similarity of physical principles, because optimization of particle trajectories (for energetically profitable) in cooperative particle dynamics as a physical basis of ferrofluid behaviour under magnetic field provides applicability of a similar approach (known as particle swarm optimization) to computer algorithms for mathematical calculations in this area.

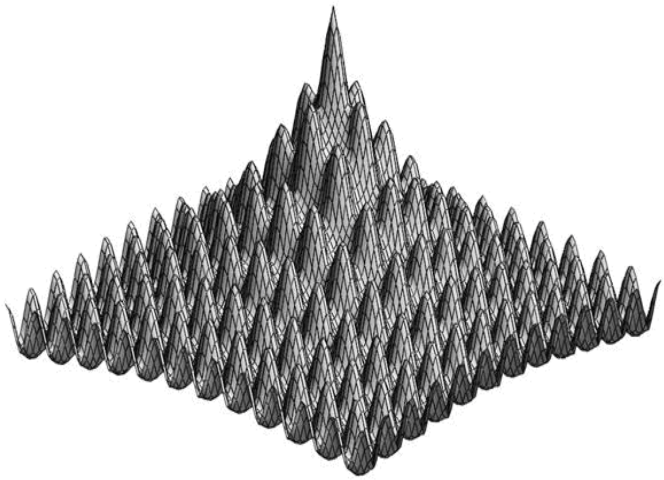

Figure 1. Griewank function (Mathcad visualization).

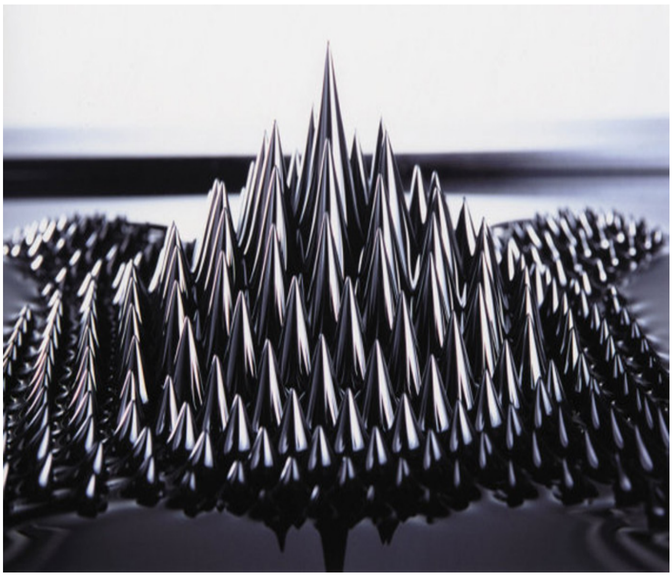

Figure 2. Example of a ferrofluid sufrace.

Another example of a forecited approach is illustrated in Fig. 3 and Fig. 4 in morphological comparison. The nature of the approximation is illustrated in Fig. 3 and the photo of ferrofluid surface with the normal-field instability is given in Fig. 4.

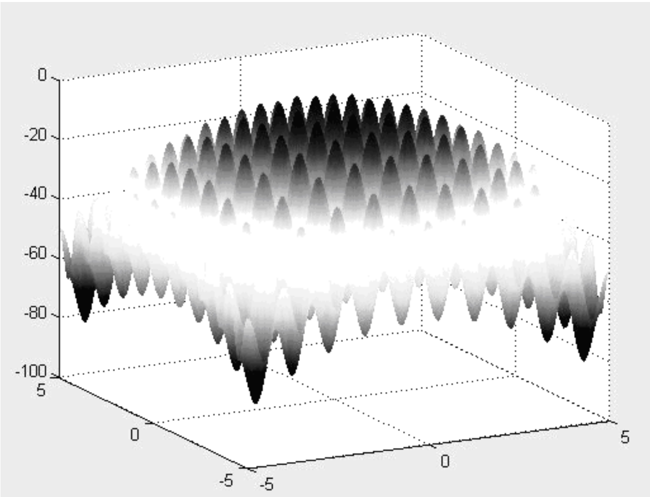

Figure 3. Another example of a Griewank function visualization.

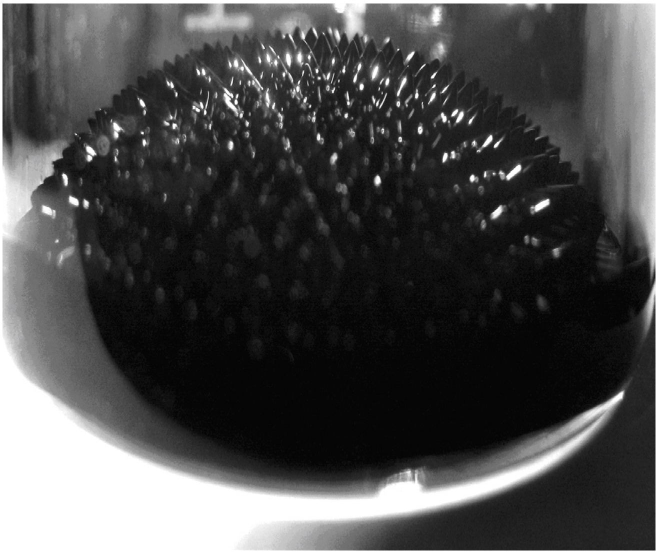

Figure 4. Another example of a ferrofluid surface.

Thus we have focused our attention at physical mechanisms of ferrofluid self-organization based on field-induced [31] collective (multi-agent [32, 33]) behavior of magnetic particles in strong magnetic fields. Strictly speaking, such a definition does not make sense because it is very obvious. We therefore limit ourselves to the most simple case, which is morphologically observable. We have not paid much attention to some more complicated situations, but it's clear that the viewpoint adopted in this article possesses a more wide application than we have already mentioned here. Our next step was to apply this idea for modelling of biomimetic pattern formation [26, 34] and a corresponding article has just been submitted to another biological journal. It was not our purpose to give a comprehensive development of the idea proposed, so in this article we have only laid a theoretical foundation for its further application. Actually, we are also little concerned about our priority, because this complex problem is still fairly difficult to be solved without collective efforts.

Discussion

We have taken a number of algorithms and MATLAB codes as a starting point for our findings. Early investigators have used various approaches for Griewank function visualization, such as given below for C++ (by M. Clerck [35]):

```
E=exp(1); two_pi=2*acos(-1);
sum1=0;sum2=0;
for (d=0;d<D;d++) {xd=x.x[d]; sum1=sum1+xd*xd;
sum2=sum2+cos(two_pi*xd);}
f=(-20*exp(-0.2*sqrt(sum1/(double)D))-exp(sum2/(double)
D)+20+E);
```

or (by Zabinsky, Khompatraporn and Ali [36]):

```
float fvalue;

   fvalue = 0.0;

   float gvalue=0.0;
   float hvalue =0.0;
   for(int index = 0 ; index<dimension ; index ++)
   {
      gvalue = gvalue + pow( *(position+index),2.0 );
      hvalue = hvalue + cos(*(position + in-
dex)*2*3.14159265359);
   }
   fvalue = -20 exp(-0.0.2 * pow((gvalue/dimension),0.5) ) -
exp(hvalue/dimension)+20+exp(1);

   return fvalue;
};
```

and their analogues for MATLAB:

```
function z = ft_ackley(x,y)
a =   20;
b =  0.2;
c = 2*pi;
d =  5.7;
f =  0.8;
n =    2;
z = (1/f)*( -a*exp(-b*sqrt((1/n)*(x.^2+y.^2))) - ...
    exp((1/n)*(cos(c*x) + cos(c*y))) + ...
    a + exp(1) + d);
```

We have widely used these or similar algorithms and codes in our computational practice, but have not recently appealed to C++. In recent years several authors (except us) unfortunately no longer use this procedure [37].

The method described, however, is fairly difficult in application, because particle swarm optimization (in general case; for most difficult problems) is a very time-consuming approach mostly requiring powerful mainframes and supercomputers. It can be done rather easily, however, if we introduce this approach in the everyday routine educational practice for ferrofluidics and material science teaching [38].

Conclusion

We have proposed a novel method for approximation of ferrofluid surface morphology formed under strong magnetic fields due to normal-field instability. These results can easily be described in terms of particle swarm optimization theory. Such elementary cases can be covered by the general Griwenk equation. The foregoing results are a very brief and simplified implementation of this basic idea, because in general case the described procedure itself is known in PSO, but its application to ferrofluidics is our contribution. The described approach is expected to possess a wide range of potential applicability in ferrofluid science.

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